Counting States of Extremal Black Holes

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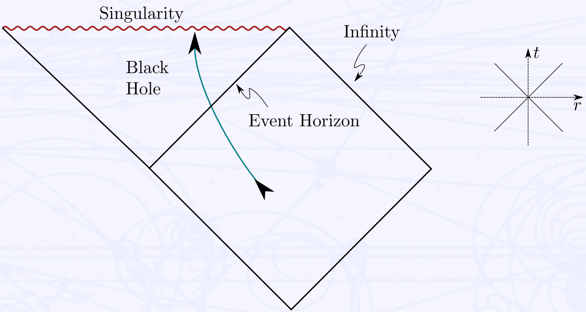
December 16th, 2010

Black Holes

Causal Diagram:



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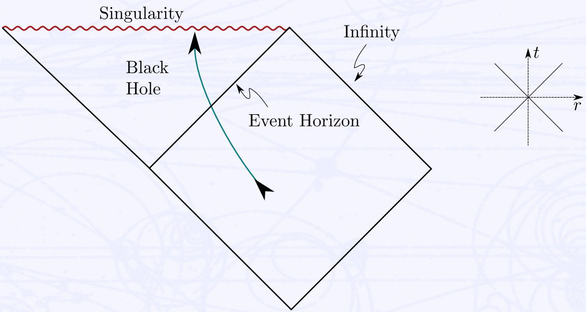

Strange entities

$$S_{\text{bh}} = \frac{A}{4G_N} \tag{1}$$

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- Standard Result in QFT in curved spaces.
- "Thermalizes" with any form of matter.
- No universal microcanonical interpretation.

Laws of Black Hole Thermodynamics

Zeroth Law	κ is constant over the horizon.
First Law	$\delta M = \frac{\kappa}{4G_N}\delta A + \Omega_H \delta J$
Second Law	$S = \frac{1}{4G_N}A$ does not decrease.
Third Law	$\kappa = 0$ can't be reach by classical processes.

Goal: to understand these laws microscopically.

The BTZ Black Hole

$$ds^2 = -\left(-M + \frac{r^2}{\ell^2} + \frac{J^2}{4r^2}\right)dt^2 + \frac{dr^2}{-M + \frac{r^2}{\ell^2} + \frac{J^2}{4r^2}} + r^2\left(d\varphi - \frac{J}{2r^2}dt\right)^2 \tag{3}$$

Solution of Einstein's Equations with a cosmological constant in 3d. Locally,

$$R_{abcd} = \Lambda(g_{ac}g_{bd} - g_{ad}g_{bc}) \tag{4}$$

Maximally symmetric! (Weyl tensor has to vanish in 3d):

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In fact, upon changing of coordinates:

$$\log(x \pm y) = \frac{1}{2} \log\left(\frac{r^2 - r_+^2}{r^2 - r_-^2}\right) + \frac{r_+ \mp r_-}{\ell}\left(\varphi \pm \frac{t}{\ell}\right), \quad z = \left(\frac{r_+^2 - r_+^2}{r^2 - r_-^2}\right)^{1/2} \exp\left(\frac{r_+}{\ell}\varphi - \frac{r_-}{\ell^2}t\right) \qquad (5)$$

with $M = r_+^2 + r_-^2, J = 2r_+r_-$. One recovers the AdS_3 metric:

$$ds^2 = \frac{\ell^2}{z^2}\left(dx^2 - dy^2 + dz^2\right), \quad (z > 0). \qquad (6)$$

But with "strange" $t - \varphi$ identifications.

Puzzling solution: can be seen as a flat connection in a $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ WZW model:

$$S = \frac{k}{4\pi} \int \mathrm{Tr}(A_+ \wedge dA_+ + \tfrac{2}{3} A_+ \wedge A_+ \wedge A_+) - (+ \to -) \qquad (7)$$

where $A_\pm = (\varepsilon^{ijk} \omega_{jk} \pm \frac{1}{\lambda} e^i) \tau_i$.

Solution $A_\pm = g^{-1} dg$. Identification $g \sim h_1 g h_2$.

Space-time Virasoro Algebra (Brown-Henneaux)

Central charge arises in classical sense. Situation akin to conformal anomaly in 2d. Regularization of a CFT with central charge c coupled to a curved background shows that

$$\langle T^a{}_a \rangle = \frac{c}{24\pi} R \tag{8}$$

This effect can be described in an "effective" action sense, by considering the effective T_{ab}:

$$\tilde{T}_{zz} = T_{zz} + \frac{c}{12\pi}[(\partial\phi)^2 - \partial^2\phi] \tag{9}$$

where ϕ is the Liouville field: $g_{ab} = e^{2\phi}\eta_{ab}$. Improvement term $\gamma R(\eta)\phi$ upsets the Poisson brackets ϕ stress-energy's Fourier coefficients satisfy:

$$\{L_n, L_m\}_{\text{P.B.}} = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m} \tag{10}$$

where $c = 1 + 6\gamma^2$. Effect of quantization is $1/\gamma \to 1/\gamma + \gamma$.

ϕ parametrization of the Virasoro algebra is called the "Liouville Sector" of the CFT. Situation quite similar to BTZ.

BTZ in string theory: D1-D5 system

N_1 D1s along the 5th dimension, N_5 D5s along the 5th-10th momentum, all compactified, and with momentum along the 5th dimension:
Near horizon metric has $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$ structure

$$ds^2 = \frac{r^2}{\ell^2}(-dt^2 + dx_5^2) + \frac{\ell^2}{r^2}dr^2 + \ell^2 d\Omega_3^2 + \left(\frac{Q_1}{Q_5}\right)^{1/2} ds_{T_4}^2 \qquad (11)$$

Same metric structure as BTZ, low lying spectrum moduli of branes: $(4+2)N_1N_5$ gauge invariant ones. Current sector can be understood as (asymptotic) space-time coordinate transformations, like ($w^{\pm} = \pm t + x_5$):

$$\delta w^+ = \epsilon(w^+), \qquad \delta r = -\frac{r}{2}\partial_+\epsilon, \qquad \delta w^- = -\frac{1}{2r^2}\partial_+^2\epsilon \qquad (12)$$

Central charge: Brown-Henneaux. Density of states can be computed by Cardy's formula:

$$S = 2\pi\left(\sqrt{\frac{cL_0}{6}} + \sqrt{\frac{c\bar{L}_0}{6}}\right), \qquad c = \frac{3\ell}{2} \qquad (13)$$

Lessons

- First microscopic description of a black hole.
- Constuction heavily dependent on SUSY.
- Interpretation of microstates as large diffeos.

Any extensions?

Bardeen: A Kerr Black hole (4d), a solution with 2 Killing vector fields:

$$\xi^a = \left(\frac{\partial}{\partial t}\right)^a, \qquad \varphi^a = \left(\frac{\partial}{\partial \varphi}\right)^a \tag{14}$$

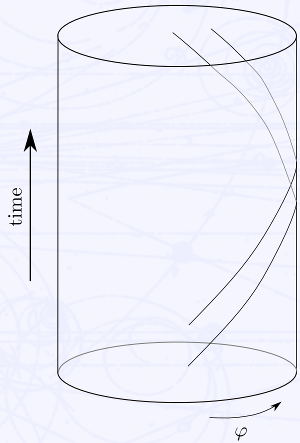
in the extremal limit $J \to M^2$, has a near-horizon limit with a $\mathrm{AdS}_2 \times \mathrm{X}$ structure:

$$g_{tt} \approx (r - r_0)^2 \tag{15}$$

Guica *et al.*: Entropy can be described by large diffeos of the form

$$\xi = \epsilon(w)\partial_w - \epsilon'(w) r \partial_r \tag{16}$$

CdC & Queiroz 1006.0510: large diffeos upsets the asymptotic $\mathrm{AdS}_2 \times \mathrm{X}$ structure but leave flat space charges invariant.


time
φ

Generic Feature

Extremal BTZ:

$$ds^2 = -\frac{(r^2 - r_+^2)^2}{r^2} dt^2 + \frac{r^2 dr^2}{(r^2 - r_+^2)^2} + r^2 \left(d\varphi - \frac{r_+^2}{r^2} dt \right)^2 \qquad (17)$$

Make $r = r_+ + 2\rho$, $\rho \ll r_+$. Lapse function and angular velocity become:

$$N(\rho) = \rho^2 - \frac{\rho^3}{r_+} + \ldots, \qquad \Omega = 1 - \frac{\rho}{r_+} + \frac{3\rho^2}{4r_+^2} + \ldots \qquad (18)$$

CdC & Queiroz: Entropy counts contact structures in the $\{t, r, \varphi\}$ submanifold. Contact structure given by

$$\alpha = d(\varphi - t) + \frac{\rho}{r_+} dt, \qquad (19)$$

restriction to $t - r$ plane gives symplectic (Kähler).
But uplift to 4d also gives symplectic structure!

Generalization

Generic Extremal $\kappa = 0$ metric:

$$ds^2 = \Omega^2(x^i)\left(-\cosh^2 z\, dt^2 + dz^2 + \Lambda^2(x^i)\,(d\varphi + \sinh z\, dt)^2\right) + \tilde{g}_{ij} dx^i dx^j. \tag{20}$$

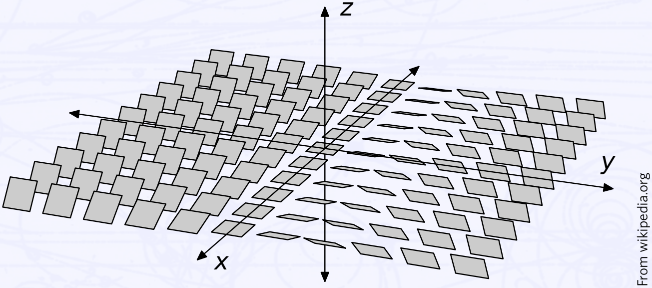
The "contact structure" $\alpha = \Omega^2\Lambda^2(d\varphi + \sinh z\, dt)$ is used to compute the angula momentum via:

$$J = \int_\Sigma {}^* d\alpha \tag{21}$$

The relevant bit is the volume form in the $r - t$ plane:

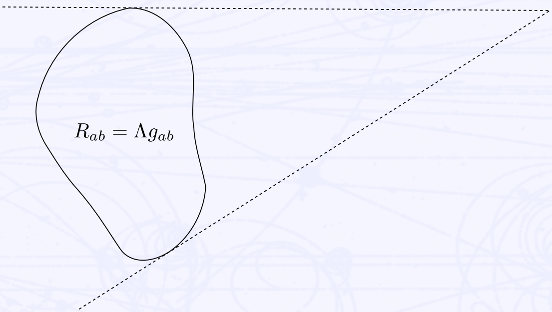
$$d\alpha = \Omega^2\Lambda^2 \cosh z\, dz \wedge dt + \text{transverse terms.} \tag{22}$$

Under Brown-Henneaux, $\alpha \to \epsilon' \alpha$. Defines an *equivalent* contact structure! (Defines the same contact hyperplanes $\mathcal{H}$ such that $\alpha_{\mathcal{H}} = 0$).



Contact planes from the canonical Darboux contact form $dz - ydx$.

Structure generalizes the Reeb vector for Sasakian manifolds. A Sasakian manifold serves as base for a cone which is a Kähler structure and the Reeb vector lifts to the Kähler form on the cone.



Sasaki manifolds with Einstein metrics are lifted to Ricci flat Kähler (Calabi-Yau) manifolds. There are important backgrounds for strings which preserves (some) SUSY.

Embedding to Strings

Near horizon extreme Kerr metric can be written as a squashed 3-sphere, fibered over the angular variables:

$$ds^2 = -\frac{\Omega^2}{2}\left[\mathrm{Tr}(g^{-1}dgg^{-1}dg) + (\Lambda^2 - 1)J_3\bar{J}_3\right] + \ldots \tag{23}$$

σ-model: Contact structure serves as B-field $=$ Chiral model. Relation to Gödel universe (Compère *et. al.* 0808.1912). Also gives central charge, when integrated over the angular, "adiabatic" variables (except φ).

IR fixed point of a marginal chiral deformation?

Special Holonomy

Underlying structure that makes identification works: Special Holonomy?
Special Holonomy vs. SUSY.
Special Holonomy vs. pp-waves.
In 4d counted by principal null spinors.

$$\Psi_{\alpha\beta\gamma\delta} = (\psi_+)_{(\alpha}(\psi_+)_\beta(\psi_-)_\gamma(\psi_-)_{\delta)} \rightarrow \Psi_{\alpha\beta\gamma\delta}\psi_\pm^\alpha\psi_\pm^\beta\psi_\pm^\gamma = 0 \qquad (24)$$

Generalization of the integrability condition for covariant spinors? Tool to bridge between weakly and strongly coupled limits?

Conclusions

- Many Black Holes microscopics are sucessfully modelled by strings.
- Major tool to understand gauge/gravity correspondence.
- Volume minimization = Zamolodchikov's theorem?
- Zero temperature: conformal fixed point of dual theory in the infrared.
- Also, have diffeomorphism interpretation of degrees of freedom.
- Embedding into strings seems feasible. BH d.o.f.'s as excitations of a chiral sector of strings.
- Search for new anomaly-free chiral perturbations of the string σ-model.

Obrigado!