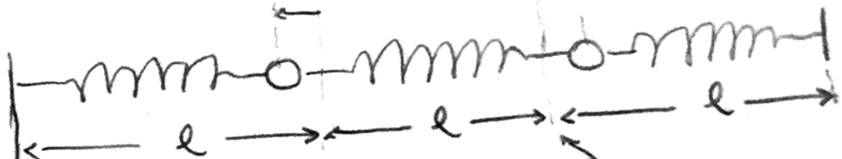
FI-595 - Mecânica Clássica 2

1º E.E. - Gabarito

Problema 1: (a) A lagrangeana é

$$L = \frac{M\dot{x}_1^2}{2} + \frac{M\dot{x}_2}{2} - V(x_1, x_2)$$

onde $V(x_1, x_2)$ é a soma das distenções de cada mola:



$$V(x_1, x_2) = \frac{k(x_2-\ell)^2}{2} - \frac{k\ell^2}{2} + \frac{k(\ell+x_2+x_1)^2}{2} - \frac{k\ell^2}{2}$$

$$+ \frac{k(\ell-x_1)^2}{2} - \frac{k\ell^2}{2}$$

$$= kx_1^2 + kx_1x_2 + kx_2^2$$

(b) As e.d.M. são

$$M\ddot{x}_1 = -2kx_1 - kx_2$$

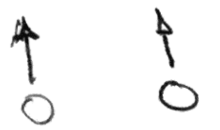
$$M\ddot{x}_2 = -kx_1 - 2kx_2$$

introduzindo $q_+ = x_1 + x_2$ e $q_- = x_1 - x_2$, temos

$$M\ddot{q}_+ = 3kq_+ \qquad \omega_+ = \sqrt{\frac{3k}{M}}$$

$$M\ddot{q}_- = kq_- \qquad \omega_- = \sqrt{\frac{k}{M}}$$

(c) Como há mais 2 graus de liberdade, temos mais 2 modos normais:

 e 

Problema 2: (a) $\vec{p} = \dfrac{\partial L}{\partial \dot{\vec{r}}} = M\dot{\vec{r}} - \dfrac{e}{2c}(\vec{B}\times\vec{r})$

e assim:

$$H = \vec{p}\cdot\dot{\vec{r}} - L = M\dot{\vec{r}}^2 - \frac{e}{2c}(\vec{B}\times\vec{r})\cdot\dot{\vec{r}}$$

$$- \frac{M}{2}\dot{\vec{r}}^2 + \frac{e}{2c}(\vec{B}\times\vec{r})\cdot\dot{\vec{r}}$$

$$= \frac{M}{2}\dot{\vec{r}}^2 = \frac{1}{2M}\left(\vec{p} + \frac{e}{2Mc}(\vec{B}\times\vec{r})\right)^2$$

(b) $$\dot{\vec{r}} = \frac{\partial H}{\partial \vec{p}} = \frac{1}{M}\vec{p} + \frac{e}{2M^2c}(\vec{B}\times\vec{r})$$

$$\dot{\vec{p}} = -\frac{\partial H}{\partial \vec{r}} = -\frac{e}{2M^2c}\left(\vec{p} + \frac{e}{2Mc}(\vec{B}\times\vec{r})\right)\times\vec{B}$$

Escolhendo coordenadas cartesianas e $\vec{B} = B\hat{z}$

$$\dot{x} = \frac{1}{M}p_x - \frac{e}{2M^2c}By \qquad \dot{p}_x = -\frac{eB}{2m^2c}\left(p_y + \frac{e}{2mc}Bx\right)$$

$$\dot{y} = \frac{1}{M}p_y + \frac{e}{2M^2c}Bx \qquad \dot{p}_y = -\frac{eB}{2m^2c}\left(-p_x + \frac{e}{2mc}By\right)$$

$$\dot{z} = \frac{1}{M}p_z \qquad \dot{p}_z = 0$$

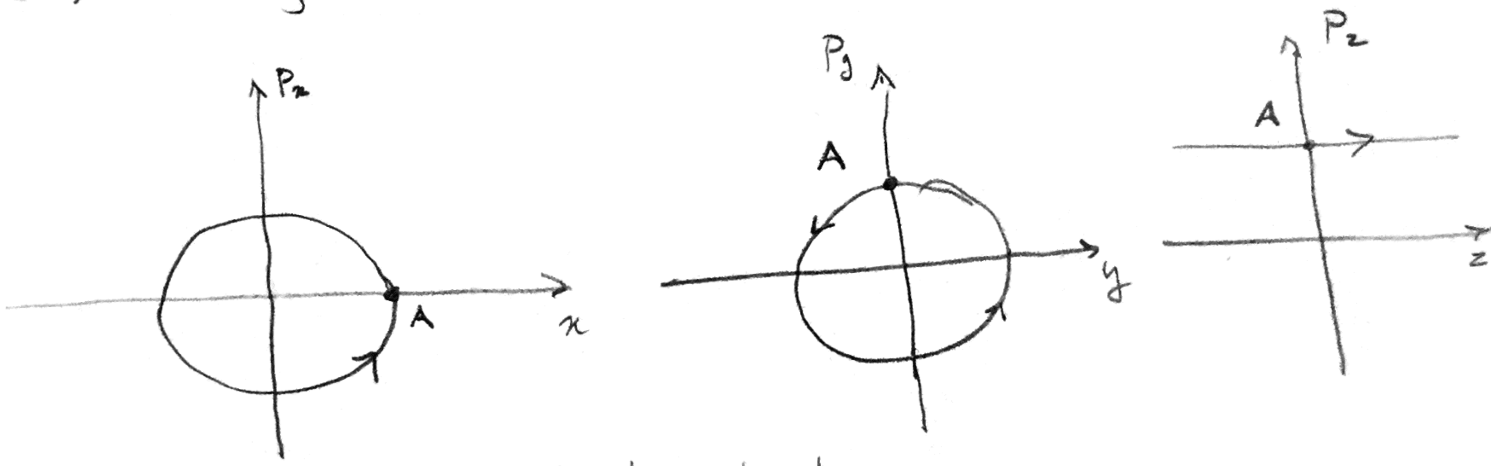
que resultam em

$$\dot{\vec{r}}_\perp = \frac{e}{m^2 c}\vec{B}\times\vec{r}_\perp$$, onde $\vec{r}_\perp = x\hat{x}+y\hat{y}$ é transverso a $\vec{B}$

e

$$\ddot{\vec{r}}_\parallel = 0$$, um movimento livre na direção paralela a $\vec{B}$.

(c) A trajetória será assim



A é o mesmo instante de tempo.

<u>Problema 3</u>: Escreva a solução particular como $x_p(t) = \sum_{n=-\infty}^{\infty} x_n e^{\frac{2\pi i t}{T}n}$

a equação para x_n se torna em

$$\sum_{n=-\infty}^{\infty}\left[-\left(\frac{2\pi}{T}\right)^2 n^2 + 2i\beta\frac{2\pi}{T}n + \omega_0^2\right]x_n e^{\frac{2\pi i t}{T}n} = \frac{F_0}{2\pi}\sum_{n=-\infty}^{\infty} e^{\frac{2\pi i t}{T}n}$$

ou seja $$x_n = -\frac{F_0}{2\pi}\left(\frac{2\pi}{T}\right)^2 \frac{1}{n^2 - \frac{2i\beta T}{2\pi}n - \frac{T^2\omega_0^2}{(2\pi)^2}}$$

$$= -\frac{2\pi F_0}{T^2}\frac{1}{\left(n-\frac{T}{2\pi}(\omega+i\beta)\right)\left(n-\frac{T}{2\pi}(-\omega+i\beta)\right)}$$

onde $\omega^2 = \omega_0^2+\beta^2$.

usando $\dfrac{1}{(n-b_1)(n-b_2)} = \dfrac{1}{b_1-b_2}\left(\dfrac{1}{n-b_1} - \dfrac{1}{n-b_2}\right)$, temos

$$x_p(t) = -\frac{2\pi^2 F_0}{T^3\omega}\sum_{n=-\infty}^{\infty} \frac{e^{\frac{2\pi i t}{T} n}}{n - \frac{T}{2\pi}(\omega + i\beta)} - \frac{e^{\frac{2\pi i t}{T} n}}{n - \frac{T}{2\pi}(-\omega + i\beta)}$$

$$= -\frac{2\pi^2 F_0}{T^3\omega}\left[\frac{2\pi i\, e^{-\beta t + i\omega t}}{1 - e^{-\beta T + i\omega T}} - \frac{2\pi i\, e^{-\beta t - i\omega t}}{1 - e^{-\beta T - i\omega T}}\right]$$

$$= \frac{8\pi^3 F_0}{T^3\omega}\,\frac{e^{-\beta t}\operatorname{sen}(\omega t) - e^{-\beta(t+T)}\operatorname{sen}\omega(t-T)}{1 - 2e^{-\beta T}\cos(\omega T) + e^{-2\beta T}}$$